Short-term profit maximization

A firm operating in a perfectly competitive market has the following short-term total cost function: $C(q) = \frac{1}{3}q^3 - 3q^2 + 19q + a$. Where a is a positive constant, and p being the price at which it sells:

1. Find the first-order condition derived from maximizing profit with the given cost function.
2. Which values of p allow for plausible values of optimal production quantities?
3. What condition must be met for it to be optimal to produce at q obtained in the maximization problem and rule out the case where it is preferable not to enter the market?
4. If $p = 19$ and $a = 37$, how much will it be convenient for the company to produce? What will be the profit?
5. Now assume that $p = 46$ and plot the profits as a function of a. For what values of a will it be convenient for the firm to produce? For what values of a will the firm have extraordinary or null profits?

Solution

1. We set up the profits:
$$B = R - C = pq - (\frac{1}{3}q^3 - 3q^2 + 19q + a)$$
With the first-order condition:
$$B'_q = p - (q^2 - 6q + 19) = 0$$
$$p = q^2 - 6q + 19$$

2. $$q^2 - 6q + 19 - p = 0$$
Using the quadratic formula to find the roots:
$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - 4(19 - p)}}{2(19 - p)} = \frac{6 \pm \sqrt{4p - 40}}{2}$$
So assuming that $4p \geq 40$, or $p \geq 10$, we have two situations:
$$q = \frac{6 + \sqrt{4p - 40}}{2}$$
and
$$q = \frac{6 - \sqrt{4p - 40}}{2}$$
As $q > 0$ is also required, we have:
$$\frac{6 + \sqrt{4p - 40}}{2} > 0$$
This condition is always met as $\sqrt{4p - 40} \geq 0$. For the other root:
$$\frac{6 - \sqrt{4p - 40}}{2} > 0$$
Solving for p, we find $p < 19$. Thus, for $10 \leq p < 19$, the solution is:
$$\frac{6 \pm \sqrt{4p - 40}}{2}$$
Whereas for $19 < p$, the solution is:
$$\frac{6 + \sqrt{4p - 40}}{2}$$

3. Production doesn't occur if:
$$p < \text{Minimum average variable cost}$$
This is the short-run shutdown condition. The average variable cost is: $\frac{1}{3}q^2 - 3q + 19$. To find its minimum, we differentiate and set equal to 0, yielding:
$$\frac{2}{3}q - 3 = 0$$
$$q = 9/2$$
Substituting this into the average variable cost function gives the shutdown price:
$$AVC(9/2) = \frac{1}{3}(9/2)^2 - 3(9/2) + 19 = 12.25$$
Thus, for any $p < 12.25$, the firm will not enter the market.

4. If $p = 19$ and $a = 37$, the possible optimal quantities are:

$$q_1 = \frac{6 + \sqrt{4 * 19 - 40}}{2} = 6$$

and

$$q_2 = \frac{6 - \sqrt{4 * 19 - 40}}{2} = 0$$

This leads to two possible profits:

$$B_1 = 19 * 6 - (\frac{1}{3}6^3 - 3 * 6^2 + 19 * 6 + 37) = -1$$

$$B_2 = 0 - 37 = -37$$

We choose the quantity q_1 because it provides a greater profit. This might seem counterintuitive because they are negative profits. However, this is because fixed costs are very high. In the short run, the company will decide to produce to cover part of these fixed costs. But in the long run, it should be able to adjust these fixed costs, or it would have to exit the market.

5. If $p = 46$, the potential optimal quantities are:

$$q_1 = \frac{6 + \sqrt{4 * 46 - 40}}{2} = 9$$

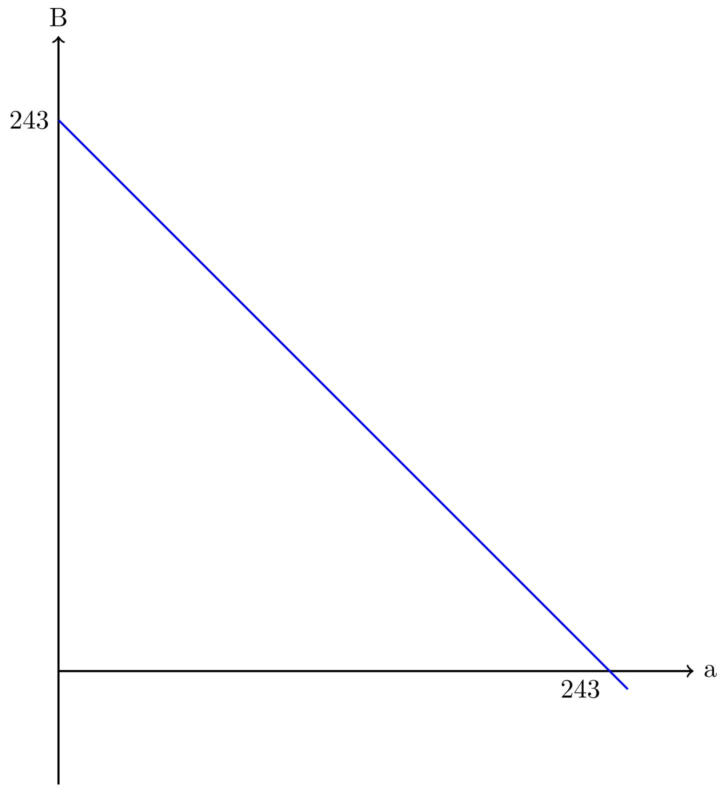
and

$$q_2 = \frac{6 - \sqrt{4 * 19 - 40}}{2} = -3$$

We choose q_1 since negative production quantities are not feasible. This leads to the profit:

$$B = 46 * 9 - (\frac{1}{3}9^3 - 3 * 9^2 + 19 * 9 + a)$$

$$B = 243 - a$$



For values of a greater than 243, the function shows null profits, and for values of $a \in [0, 243]$ the profits are null or positive. As in the short run we cannot modify fixed costs, the firm will produce for any value of a because even if it cannot have positive profits, it can at least cover part of the fixed costs. This is as long as the price is higher than the previously calculated shutdown point.